Coordinating monetary and macro-prudential policies*

Ambrogio Cesa-Bianchi† Alessandro Rebuucci‡

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Abstract

We develop a simple model featuring both a macroeconomic and a financial stability objective that speaks to the interaction between monetary and macro-prudential policies. First, we find that real interest rate rigidities have an asymmetric impact on financial stability, by exacerbating the effects of financial frictions in response to expansionary shocks to the economy, while acting as an automatic macro-prudential stabilizer in response to contractionary shocks. Second, when the policy interest rate is the only available instrument, a policy authority subject to the same constraints as those of private agents cannot always achieve a (constrained) efficient allocation and faces a trade-off between macroeconomic and financial stability. This has important implications for the role played by the Federal Reserve in the run up to the global financial crisis. Our simple model suggests that the weak link in the policy framework seems not to be an excessively lax monetary policy stance, but rather the absence of a second policy instrument aimed at preserving systemic financial stability.

Keywords: Monetary Policy, Macro-Prudential Policies, Financial Crises, Real Rigidities, Credit Frictions.

JEL code: E44, E52, E61.

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†Università Cattolica di Milano and Inter-American Development Bank. Correspondence to: Research Department, Inter-American Development Bank, 1300 New York Avenue, 20577, Washington DC, USA. Email: ambrogio.cesabianchi@gmail.com

‡Inter-American Development Bank.
1 Introduction

The global financial crisis and ensuing great recession of 2007-09 have ignited a debate on the role of policies for the stability of the financial system or the economy as a whole (i.e., macro-prudential policies). In advanced economies, this debate is revolving around the role of monetary and regulatory policies in causing the global crisis and how the conduct of monetary policy and the supervision of financial intermediaries should be altered in the future to avoid a recurrence of this catastrophic event. In this paper we develop a simple model featuring both a macroeconomic and a financial stability objective that speaks to the interaction between monetary and macro-prudential policies.

The prime objective of macro-prudential policy is to limit build-up of system-wide financial risk, in order to reduce the probability and mitigate the impact of a financial crash (see Bank of England, 2009, IMF, 2011, Borio, 2011, for a discussion). At the same time, most commonly used prudential tools require an interaction between macro-prudential policy and other policies. The overlap between different policy areas is one of the major challenges for policy-makers, who have to consider the unintended impact of their instruments on other policy objectives and the unintended impact of other policy-makers’ instruments on their own policy objective (Svensson, 2012b).

On the one hand, loosely defined monetary policy might affect financial stability. For example, investors may be pushed to substitute low-yielding, safe assets for higher yielding, riskier assets (Rajan, 2005, Dell’Ariccia et al., 2011); investors may also be encouraged to take greater risks if they perceive that monetary policy is being used asymmetrically on asset prices (Issing, 2009); and asset price increases induced by falling interest rates might cause banks to increase their holdings of risky assets through active balance sheet management (Adrian and Shin, 2009, 2010). On the other hand, macro-prudential policy instruments can have an effect on macroeconomic stability. In fact, by affecting variables such as asset prices and credit, macro-prudential policy is likely to affect a key mechanism of transmission of monetary policy (see e.g., Ingves, 2011). This overlap entails the possibility of the instruments having conflicting (or amplifying) effects if they are implemented in an uncoordinated manner by authorities with different objectives, possibly leading to a worse outcome than if the instruments had been coordinated (see Bean et al., 2010, Angelini et al., 2011, between others).

Against this background, some observers have assigned monetary policy a central role in exacerbating the severity of the global financial crisis of 2007-09. In a paper that openly embraces this view, Taylor (2007) noticed that — during the period from
2002 to 2006—the U.S. Fed funds rate was well below what the rule of thumb of the previous two decades of good economic performance would have predicted. Figure 1 displays the actual Fed funds rate (solid line) and the counterfactual policy rate that would have prevailed if monetary policy followed a standard Taylor rule (dashed line).

**Figure 1** A counterfactual path for the U.S. policy rate

![Figure 1: A counterfactual path for the U.S. policy rate](image)

**Note.** This chart replicates the counterfactual Fed fund rate as presented in Taylor (2007). The counterfactual path for the policy rate is obtained with a Taylor rule of the type: \( i_t = r_t + 1.5(\pi_t - \pi) + 0.5(y_t - y^*_t) \), where \( r_t \) is set to 2 percent, \( \pi_t \) is CPI inflation, \( \pi \) is target inflation (assumed at 2 percent), \( y_t \) is real GDP growth, and \( y^*_t \) is real potential GDP growth. **Source.** FRED Economic Data.

Figure 1 shows that, in fact, the interest rate implied by the Taylor rule is well above the actual Fed fund rate, starting from the second quarter of 2002. Taylor (2007) shows that such counterfactual policy rate would have reduced the rapid growth of the housing market bubble; moreover, he supports the idea that the deviation from this rule-based framework has been a major factor in determining the likelihood and the severity of the crisis (Taylor, 2010).

Despite it is a widely shared sentiment that the Fed is partly to blame for the housing bubble and its resultant economic impact, the issue is highly controversial.\(^1\) “The best response to the housing bubble would have been regulatory, rather than monetary,” said Bernanke (2010) in the remarks to the American Economic Association’s annual meeting in Atlanta, 2010. Such disagreement is also reflected in the substantially different institutional arrangements for the implementation of macro-prudential and monetary policies that emerged in different countries after the crisis.

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\(^1\)Besides Taylor (2007, 2010), Borio and White (2003), Gordon (2005), and Borio (2006) support the idea that monetary policy contributed significantly to the financial boom that preceded the global financial crisis; in contrast, Posen (2009), Bean (2010), and Svensson (2010) provide arguments against this thesis.
At one extreme, the Bank of England has been assigned full responsibility for macro-prudential policy and monetary policy. On the other extreme, in the U.S., these functions remains relatively uncoordinated. Yet the interaction between macro-prudential and monetary policy has received surprisingly little attention in the literature.

To address the issues arising from the above discussion, we develop a simple model of consumption-based asset pricing and collateralized borrowing in which consumers hold an asset which can be used as collateral to get loans from banks. The model is a 3-period economy similar to the one analyzed in Jeanne and Korinek (2010a), extended with a stylized monopolistic competitive banking system. The presence of a real and a financial friction gives rise to both a traditional macroeconomic stabilization role for policy and a more novel financial stability objective. The macroeconomic stabilization objective arises from the presence of monopolistic competition in the banking sector and real interest rates rigidities. Due to monopolistic power, banks apply a non-trivial markup on lending rates. Moreover, when banks cannot fully adjust their lending rates in response to macroeconomic shocks, the economy displays distortions typical of models with staggered price setting, generating equilibrium allocations that are not Pareto efficient. The financial stability objective stems from the fact that the model endogenously generates financial crisis and embeds systemic risk. When access to credit is subject to an occasionally binding collateral constraint, a pecuniary externality (or price, or credit externality) arises. Atomistic agents do not internalize the effect of their individual decisions on a key market price entering the specification of the financial friction, thus driving a wedge between the competitive and the planner equilibria.

The analysis of our decentralized economy shows that real interest rate rigidities interact in an asymmetric fashion with the distortionary effects of the financial friction, depending on the sign of the shock hitting the economy. In response to positive shocks to the risk free interest rate, the real rigidity exacerbates the overborrowing induced by the financial friction —because average lending interest rates raise less than the risk free rate— thus increasing the probability of observing a financial crisis next period. In contrast, when the risk free interest rate is hit by a negative shock, the same real rigidity acts as an automatic macro-prudential stabilizer that dampens overborrowing and reduces the probability of a crisis next period —because average lending interest rates fall less than the risk free rate.

The model also shows that a single authority, obeying the same financial constraints faced by private economic agents (i.e., constrained efficient allocation) and with two different instruments (as, for example, a tax on debt and the policy interest
rate) can achieve efficiency in terms of allocation in response to both positive and negative shocks to the risk free interest rate.\(^2\) However, when only one instrument is available, achieving constrained-efficiency depends on the sign of the shock hitting the economy. In case of negative shocks to the risk free interest rate (say because of a negative demand shock), the social planner endowed with only one instrument may not be able achieve the efficient allocation.

This has important implications regarding the role played by U.S. monetary policy for the stability of the financial system in the run up to the global financial crisis. In particular, we show that Taylor’s argument —i.e., that higher interest rates would have reduced both the probability and the severity of the crisis— is supported by our theoretical model only with the auxiliary assumption that the policy authority —addressing all distortions present in our model— has just one instrument at its disposal, namely the policy rate. However, Taylor’s argument is no longer valid when the policy authority has two different instruments: in this case, in response to a negative shock, interest rates can be lowered as much as needed without concerns for financial stability.

The main message of this article can be summarized as follows. If the policy rate is the only available tool, achieving the monetary policy objectives and maintaining financial stability entails a trade off that should be taken into account by the policy authority. Specifically, the use of the policy interest rate as the only instrument to address both macroeconomic and financial frictions might lead to poorer outcomes —relative to a case in which two instruments are available— for macroeconomic and financial stability alike. Normally, however, other instruments are at policy makers’ disposal in order to achieve and maintain financial stability. Our model shows that, when two instruments are available, the trade off disappears. As suggested by Bernanke (2010) and Blanchard et al. (2010), this implies that the same monetary policy stance as the one adopted by the Fed during the 2002-06 period, accompanied by stronger regulation and supervision of the financial system, might have been more effective in reducing the likelihood and the severity of the crisis —relative to a tighter monetary policy stance with the same financial supervision and regulation observed during the 2002-06 period.

**Literature.** The ideas in this article connect with two distinct of strands of previous literature.\(^3\) The first is the branch of the New Keynesian literature that considers

\(^2\)Woodford (2011), Kashyap and Stein (2012), and Svensson (2012a) also find that, when two instruments are available, the trade off disappears and the central bank can simultaneously accomplish the goals of macroeconomic and financial stability.

\(^3\)An extensive literature review —that goes beyond the scope of the brief review presented in this section— is provided by Galati and Moessner (2011).
financial frictions and Taylor-type interest rate rules (see Angelini et al., 2011, Beau et al., 2012, Kannan et al., 2012, for example). These papers consider either interest rules augmented with macro-prudential arguments —such as credit growth or asset prices— or combination of interest and macro-prudential rules in order to allow monetary policy to “lean against financial winds”. However, in this class of models, crisis and regular business cycle are not differentiated: macro-prudential regulation is therefore taken for granted, in the sense that it does not target a well defined market failure.

The second is a growing literature that interprets financial crises as episodes of financial amplification in environments where credit constraints are only occasionally binding. In this class of models the need for macro-prudential policies stems from a fundamental market failure: a pecuniary externality originating from the presence of a key market price in the aggregate collateral constraint faced by private agents (see, between others, Benigno et al., 2012, Bianchi, 2011, Bianchi and Mendoza, 2010, Jeanne and Korinek, 2010b, Stein, 2012). However, in these models the financial friction is the only distortion in the economy. The question of how the pursue of financial stability may affect macroeconomic stability is therefore left unresolved.

Finally, few notable exceptions consider both frictions at the same time. Benigno et al. (2011) analyze a fully specified NOEM 3-periods model that features the same financial friction analyzed here and Calvo-style nominal rigidities. The solution of the fully non-linear version of the model (i.e., without resorting to approximation techniques) shows that there is a trade off between macroeconomic and financial stability but it is quantitatively small. Woodford (2012) sets up a New Keynesian model with credit frictions, where crises are endogenous but they are modelled in a relatively simple reduced-form. While silent about the endogenous mechanisms that give rise to a crisis, Woodford (2012) is able to analytically spell out an optimal target criterion according to which —under certain circumstances— the central bank may face a trade off between macroeconomic and financial stability. We should bear in mind, as it is common in this cases, that the details of the optimal policy prescription may vary with different specifications of the financial friction.

This brief review of the literature defines the contribution of the paper. On the one hand, our model features a micro founded credit friction that endogenously generates financial crises. On the other hand, it is simple enough to yield semi-closed form solutions. As a result, our analysis delivers stark implications for the discussion of monetary and macro prudential policies in the run up to the global financial crisis.

The rest of the paper is organized as follows. In section 2 we describe the model economy. Section 3 and 4 characterize the decentralized and the socially planned
equilibrium of the economy, respectively. In section 5 we discuss the implications of the model in terms of macroeconomic versus financial stability. In section 5 we conclude.

2 The model

We develop a model of consumption-based asset pricing and collateralized borrowing, in which consumers hold an asset which can be used as collateral to get loans from a stylized monopolistic banking sector. There are only three periods, denoted $t = 0, 1, 2$: the “short run”, the “medium run”, and the “long run”. The economy is populated by a continuum of atomistic identical consumers and a continuum of monopolistically competitive banks, each with a mass normalized to one.\footnote{The model can be extended to include firms which hire endogenously supplied labor by households. The set up is similar to the one used in Jeanne and Korinek (2010a).}

The model is characterized by one financial friction and two real frictions. The financial friction is given by the presence of collateralized borrowing. The first real friction is due to the presence of market power in loan markets, exercised by monopolistically competitive banks. The second distortion results from our assumption of infrequent adjustment of interest rates by banks.

In order to build intuition on the working of the model, it is useful to describe its timing. In period 0, households want to consume more than what they have and, in order to do so, they need to borrow from banks. They have a well defined demand function for loans ($b_1$) which is decreasing in the lending interest rate ($R_{L1}$). Monopolistic banks freely borrow from outside lenders at the risk free interest rate $R^*$ and, given loan demand, optimally set their lending rates. However, notice that the risk free interest rate can be hit by a temporary shock ($\pm \nu$) at the beginning of period 0. As we shall see later, we assume that only a fraction ($\mu$) of banks can re-set their lending rates conditional on the shock, while the remaining ($1 - \mu$) banks need to keep their lending rates fixed. After the realization of the shock, which is observed by all agents, the credit market clears. Notice that households also own one unity of an asset ($\theta_0$) which they either sell in order to consume or pledge as a collateral to roll over their debt in period 1. At the end of the period households consume ($c_0$).

In period 1 households receive a stochastic endowment $e$, repay their debt ($b_1 R_{L1}$), borrow an additional amount ($b_2$) from banks, realize banks profits ($\pi_1$), and consume ($c_1$). Notice that debt roll over ($b_2$) is subject to a constraint, which takes the form of $b_2 \leq \theta_1 p_1$. If hit by a shock in period 0, the level of the risk free interest rate returns
to its steady state value ($R^*$).

Period 2 represents the long run. Households get a deterministic return $y$ on the asset that they own, repay their debt ($b_2 R_{L2}$), realize banks profits ($\pi_2$), and consume ($c_2$).

### 2.1 Households

The utility of the representative consumer is given by:

$$u(c_0) + u(c_1) + c_2. \quad (1)$$

where, for simplicity, we assume a unitary discount factor. The period utility function, $u(\cdot)$, is a standard CES function:

$$u(c_t) = \frac{c_t^{1-\varphi}}{1-\varphi}. \quad (2)$$

The budget constraint can be written as:

$$\begin{cases} c_0 = b_1 + (1 - \theta_1)p_0, \\ c_1 + b_1 R_{L1} = e + b_2 + (\theta_1 - \theta_2)p_1 + \pi_1, \\ c_2 + b_2 R_{L2} = \theta_2 y + \pi_2. \end{cases} \quad (3)$$

Initially, each domestic consumer is endowed with $\theta_0 = 1$ unit of the asset. The price of the asset at time $t$ is denoted by $p_t$. Therefore, in order to consume in period 0, households need to either sell a fraction of their assets or borrow from banks and amount $b_1$. Households can buy or sell the asset in a perfectly competitive domestic market, but in a symmetric equilibrium we must have $\theta_0 = \theta_1 = \theta_2 = 1$. As we focus on symmetric equilibria, the resource constraint can be written as:

$$\begin{cases} c_0 = b_1, \\ c_1 + b_1 R_{L1} = e + b_2 + \pi_1, \\ c_2 + b_2 R_{L2} = \theta_2 y + \pi_2. \end{cases} \quad (4)$$

Each consumer, in period 1, faces a collateral constraint à la Kiyotaki and Moore (1997), which takes the form:

$$b_2 \leq \theta_1 p_1, \quad (5)$$

where $\theta_1$ is the quantity of domestic collateral held by the consumer at the beginning of period 1. The micro-foundation for the constraint is that a consumer could walk
away from his debt, following which banks could seize his asset and sell it to other consumers in the domestic market, recovering $\theta_1 p_1$.

Households maximize (1) subject to the budget constraint (4) and the collateral constraint (5). The utility maximization problem can be written as:

$$\max_{b_1, b_2, \theta_1, \theta_2} \mathcal{V} = u\left(b_1 + (1 - \theta_1)p_0\right) + \mathbb{E}\left[u\left(e + b_2 + (\theta_1 - \theta_2)p_1 + \pi_1 - b_1 R_{L1}\right)\right] + \theta_2 y + \pi_2 - b_2 R_{L2} - \lambda(b_2 - \theta_1 p_1).$$

The first order conditions read:

$$p_0 = \frac{p_1 u'(c_1) + \lambda}{u'(c_0)},$$
$$p_1 = \frac{u}{u'(c_1)},$$
$$u'(c_0) = \mathbb{E}[R_{L1} u'(c_1)],$$
$$u'(c_1) = R_{L2} + \lambda. \tag{7}$$

The first two equations represent the asset pricing conditions for the economy in period 0 and 1. The second two equations are the Euler equation of consumption in period 0 and 1. By substituting for the CES utility function, we can derive the following optimal expressions for consumption:

$$\begin{cases} c_0 = \mathbb{E}\left(R_{L1} (R_{L2} + \lambda)\right)^{-\frac{1}{\delta}}, \\ c_1 = (R_{L2} + \lambda)^{-\frac{1}{\delta}}. \tag{8} \end{cases}$$

In order to allow for market power in the banking sector, we model the market for loans in a Dixit and Stiglitz (1977) framework.\textsuperscript{5} That is, we assume that units of loan contracts bought by households are a composite constant elasticity of substitution basket of slightly differentiated financial products — each supplied by a bank $j$ — with elasticity term equal to $\zeta$ (which will be a major determinant of spreads between bank rates and the risk free rate).

In particular, the household $i$, in order to obtain a loan of a given size $b_t(i)$, needs to take out a continuum of loans $b_t(i, j)$ from all existing banks $j$, such that:

$$b_t(i) \leq \left(\int_0^1 b_t(i, j)^{\frac{\zeta - 1}{\zeta}} d_j\right)^{\frac{\zeta - 1}{\zeta}} \tag{9}$$

where $\zeta > 1$ is the elasticity of substitution between differentiated loans (or banking

\textsuperscript{5}Benes and Lees (2007) and Gerali et al. (2010) take a similar approach.
services, in general). Demand by household $i$ seeking an amount of real loans equal to $b_t(i)$ can be derived from minimizing over $b_t(i,j)$ the total repayment due to the continuum of banks $j$ (see the Appendix for a full derivation of loans demand). Aggregating over symmetric households, the minimization problem yields downward-sloping loans demand curves of the kind:

$$b_t(j) = \left( \frac{R_{Lt}(j)}{R_{Lt}} \right)^{-\xi} b_t.$$  \hfill (10)

where the aggregate interest rate on loans is given by:

$$R_{Lt} = \left( \int_0^1 R_{Lt}(j)^{1-\xi} dj \right)^{\frac{1}{1-\xi}}.$$  \hfill (11)

### 2.2 Banks

There is a continuum of monopolistically competitive domestic banks indexed by $j \in (0,1)$ owned by households. Microeconomic theory typically considers market power as a distinctive feature of the banking sector (Freixas and Rochet, 2008). In particular, we assume that each bank $j$ supplies slightly differentiated financial products, and no other bank produces the same variety: each bank has, therefore, some monopoly power over its products. However, each firm competes with all the remaining firms, since consumers consider each firm’s brand as a substitute —however imperfect— to all other available brands. As banks have market power over the supply of their products, they set prices to maximize their profits, keeping into account the elasticity of demand for their varieties.

Each bank $j$ collects fully insured deposits $d_t(j)$ from foreign investors at the natural (risk-free) interest rate $R_t = R^*$, where $R^*$ is exogenous and given. We assume further that outside lenders have infinite supply for deposits (as in Jeanne and Korinek, 2010b), so that banks can satisfy whichever demand for loans. Finally, banks use deposits to produce loans to consumers with the following constant return to scale production function:

$$b_t(j) = d_t(j)$$  \hfill (12)

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6The presence of market power can be justified by the existence of switching costs, due to asymmetric information problems which typically lead to long-term relationships between banks and borrowers (see Diamond (1984) for example). Empirically, the presence of market power in the banking sector, as well as its determinants over the business cycle, are well documented. See for example Berger et al. (2004) and Degryse and Ongena (2008).
In each period, bank $j$ maximizes its profits over both prices and quantities:

$$
\max_{R_{Lt}(j), b_t(j)} b_t(j) R_{Lt}(j) - d_t(j) R_t,
$$

subject to the demand schedule in (10) and to the production function in (12). The first order condition reads:

$$
(1 - \zeta) R_{Lt}(j)^{-\zeta} + \zeta R_{Lt}(j)^{-\zeta-1} R_t = 0,
$$

which implies that the optimal lending rate applied by banks is a positive gross markup ($M$) over the marginal cost:

$$
R_{Lt}(j) = \frac{\zeta}{\zeta - 1} R_t = MR_t. \quad (13)
$$

Notice that, together with households optimality conditions, equation (13) defines the equilibrium of the economy. That is, once the lending rate has been set by banks, households make their consumption (and, therefore, borrowing) decisions and the loans market clears.

We also assume that banks cannot fully adjust their lending rates in response to macroeconomic developments, due to interest rate stickiness. We implement this interest rate stickiness, through a one-period real rigidity.

In particular we assume that, if the risk free interest rate is hit by a temporary shock ($\nu$) in period 0, only a fraction $\mu$ of the banks can update this information by resetting their prices, whereas the remaining $1 - \mu$ banks cannot. This entails that, following a shock to the risk free interest rate, the average lending rate will be in general different from the one desired by banks: remembering that consumers are price takers and that their loans demand depends on the average interest rate in the economy, this friction will lead to a distortion in the competitive equilibrium and will create the scope for monetary policy intervention to restore efficiency. Moreover, given that the incomplete pass-through of changes in the risk free rate on lending rates is a realistic assumption just in the short run, we assume that from period 1, interest rates are again fully flexible.

Finally, notice that shocks to the interest rate in period 0 are observed by all agents before they take their optimal decisions. Therefore, such a shock does not introduce any additional stochastic element to our model economy. We will consider

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7This latter, unconventional assumption is supported by theoretical and empirical findings: for example, Gerali et al. (2010) show that the adjustment of banks lending rates to changes in the risk-free rate is only partial and heterogeneous.
three different scenarios: no shock to the risk free interest rate \((v = 0)\), a temporary increase of the risk free rate \((v > 0)\), and a temporary reduction of the risk free rate \((v < 0)\). We can consider these three scenarios as the result of a realized temporary “shock” to the risk free interest rate at the beginning of period 0.

### 2.3 Calibration

To be able to solve and simulate the model we need to make assumptions about few key parameters: the distribution of the stochastic endowment \(e\), the return of the asset \(y\), households preferences \(\rho\), the degree of monopolistic competition in the banking sector \(\zeta\), the risk free interest rate \(R^*\), the degree if stickiness of interest rates \(\mu\), and the size of the shocks to the interest rate \(v\). Table 1 summarizes the calibration of the parameter values.

<table>
<thead>
<tr>
<th>Table 1 Calibration of model’s parameter</th>
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<tr>
<td>General</td>
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<tr>
<td>Average Endowment (\bar{e})</td>
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<tr>
<td>Asset return (y)</td>
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<tr>
<td>Risk free rate (R^*)</td>
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<tr>
<td>Elasticity of Subst. (Loans) (\zeta)</td>
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<tr>
<td>Mark up (\mathcal{M})</td>
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<tr>
<td>Risk Aversion Coefficient (\rho)</td>
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<tr>
<td>Shocks</td>
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<tr>
<td>Shock to the endowment (\tilde{e})</td>
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<tr>
<td>Shock to the interest rate (v)</td>
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</table>

Notice that we solve the model for different values of the maximum size of the shock to the endowment \(\epsilon\). In particular, we will consider values of \(\epsilon\) such that the economy may be constrained for sufficiently large negative shocks but would not be constrained in the absence of uncertainty. As shown in the technical appendix, under these assumptions the model can be solved to a great extent in closed form. When a closed form solution is not available, we resort to numerical methods.

We assume that the endowment \(e\) has expected value \(\bar{e} = 1.3\) and is subject to a shock of the type:

\[
e = \bar{e} + \tilde{e},
\]

(14)

where \(\tilde{e}\) is uniformly distributed over \([-\epsilon, +\epsilon]\) —alternatively, this implies that the endowment \(e\) is uniformly distributed over the interval \([\bar{e} - \epsilon, \bar{e} + \epsilon]\). We also assume
that the deterministic return on the asset is \( y = 0.8 \). Under the current calibration, the model economy is never constrained when \( \varepsilon \leq 0.13 \). Below this threshold, the constraint never binds, the probability of a crisis is zero, and the model has a closed-form solution given by optimality conditions (7) together with \( \lambda = 0 \). In contrast, when \( \varepsilon > 0.13 \) there exist a positive probability that the constraint will bind: in this case the model does not have a closed-form solution and, therefore, the level of debt and consumption have to solved numerically (as shown in Appendix A.3).

The gross risk free interest rate is set to \( R^* = 1.03 \); then, we set the elasticity of substitution between financial products to \( \zeta = 52.5 \), which implies a gross markup of \( M \approx 1.02 \). The markup is calibrated to yield approximately a spread of 200 basis points over the risk free interest rate, which is broadly consistent with households loans data in the U.S. over the period 1999-2007.\(^8\) Household preferences are given by a constant elasticity of substitution utility function, with a relative risk aversion coefficient \( \varrho = 2 \), as is conventional in most DSGE models.

Concerning the degree of interest rate stickiness, we assume that only 50 percent of the banks that can adjust their lending rates, conditional to a movement in the interest rate. Finally, we assume that risk free interest rate is affected by a shock in period 0, such that:

\[
R_1 = R^* + \upsilon, \tag{15}
\]

where \( \upsilon \) can take three values, namely \( \upsilon = \{0, +2\%, -2\%\} \).

### 3 Equilibrium and model dynamics in the decentralized economy

We can now analyze the model and its properties, starting by characterizing the decentralized equilibrium of this economy. In order to build intuition on its working, we will consider first the effects of the financial friction —which manifests itself conditional on shocks to the endowment— by comparing the optimal allocations of our model economy with the allocations in an economy where collateral constraint is never binding. Second, we will analyze the effect of the macro friction —which manifests itself conditional on shocks to the risk free interest rate— by comparing the optimal allocations in our model economy with the allocations in an economy

\(^8\)Notice here that Gerali et al. (2010) set the elasticity of loan contracts to about 2.5, to match an average spread of 170 basis points of deposit rates on policy rate. Our number differs from theirs simply because we assume that the markup is applied to gross interest rate (i.e., \( MR \)) instead of the net interest rate (i.e., \( 1 + MR \)).
with fully flexible interest rates. Third, and finally, we will analyze the full model, i.e. when both frictions at work simultaneously.

3.1 The financial friction

As noted above, the only stochastic element in our model economy is the endowment \( (e) \) received by households in period 1, which is hit by a shock \( \tilde{\varepsilon} \) with maximum size \( \varepsilon \). As explained by Jeanne and Korinek (2010a), the financial friction affects the economy depending on whether the collateral constraint is binding or not: if the realization of the shock \( \tilde{\varepsilon} \) is large enough to make the collateral constraint to be binding, the model economy experiences an adverse feedback loop. That is, when the borrowing constraint binds, consumption declines, thus reducing asset prices and tightening the borrowing constraint. Therefore, the shock leads to a downward spiral of declining consumption, dropping asset, and tighter borrowing constraints typical of “financial accelerator” models (as, for example, in Bernanke et al. (1996) and Kiyotaki and Moore (1997)).

To fully understand the effects of the financial friction, and explained in the calibration section, we consider different values of the maximum size of the shock such that 1) the collateral constraint never binds, i.e. the shock \( \tilde{\varepsilon} \) is not large enough to push the economy in the constrained region; and 2) the collateral is occasionally binding, i.e. for large enough realizations of the shock \( \tilde{\varepsilon} \) the economy may enter the constrained region and experience a crisis.

Figure 2 displays the level of borrowing \( (b_1) \) and the crisis probability \( (\pi) \), namely the probability of the constraint to be binding, for different values of the maximum size of the shock. The figure shows that, depending on \( \varepsilon \), the economy is either never constrained or constrained with positive probability. In particular, when the constraint is never binding, households’ borrowing is not affected by the size of the shock \( \varepsilon \). In contrast, when there is a positive probability that the constraint will bind, households optimally reduce their credit demand. Moreover, notice that the amount of borrowing decreases — and the probability of a crisis \( (\pi) \) increases — in a non linear fashion with the maximum size of the shock to the endowment.

The intuition for the results in Figure 2 is the following. The Lagrangian multiplier \( \lambda \) in the Euler equation of consumption (7), represents the shadow cost of the collateral constraint. When the shock to the endowment is not large enough to push the economy in the constrained region, \( \lambda = 0 \) and the economy achieves its first-best allocation. In contrast, when the shock is large enough, \( \lambda \) is positive and increasing in the maximum size of the shock. Therefore, the larger the shock, the larger is the
Figure 2 Model equilibrium with financial friction

Note. On the horizontal axis is the maximum size of the endowment shock (\( \epsilon \)). The left-hand panel displays households’ borrowing in period 0 (\( b_1 \)), while the right-hand panel displays the implied crisis probability (\( \pi \)), that is the ex-ante probability that the constraint would bind in period 1.

value of \( \lambda \), and the smaller is optimal consumption (and borrowing) in period 0.

3.2 The macroeconomic friction

We can now switch to the analysis of the how the macro friction affects our model economy. As is well known from the standard New Keynesian literature, there are two potential distortions typical of models with monopolistic competition and staggered price setting. First, monopolistic power forces average output below the socially optimal level (as we shall see in the next section). Second, nominal rigidities generate time variations in price markups which, in turn, imply an inefficient allocation of resources. Our model displays both distortions.

As noted above, the effects of the macroeconomics friction are evident conditional on a shock to the risk free interest rate, as defined in equation (15). The shock \( \nu \) can be interpreted as a global demand shock —such as a preference shock or a government spending shock— for a small open economy (see Adolfson et al. (2007) and Harrison and Oomen (2010) for an example).

Let’s assume for the moment that interest rates are free to adjust and that lending rates at the beginning of period 0 are set to the desired optimal level, namely a markup over the marginal cost (\( R_{L1} = MR^* \)). If a positive shock \( \nu > 0 \) hits the economy,
banks face a new, higher marginal cost and update their lending interest rates such that \( R_{L1} = M(R^* + \upsilon) \). Households update their loans demand accordingly and the loans market clears: in response to the higher interest rate, consumption and borrowing in period 0 fall relative to the case in which \( \upsilon = 0 \). This allocation (henceforth “flex-rates” allocation) is efficient conditional on the shock.

However, in a sticky-rate environment, not all banks can reset their lending rate as to be consistent with the new marginal cost. The fraction \( \mu \) of banks that can reset lending rates will clearly set:

\[
R_{L1}^\mu = M(R^* + \upsilon),
\]

In contrast, the remaining \( 1 - \mu \) banks will not be allowed to reset their lending rates even though their marginal cost has changed, implying that they are forced to apply a non optimal markup:

\[
R_{L1}^{1-\mu} = M(R^* + \upsilon) < R_{L1}^\mu,
\]

with \( \widetilde{M} < M \).\(^9\) As a consequence, the average lending rate in the economy can be computed as:

\[
\bar{R}_{L1} = M(R^* + \mu \upsilon),
\]

which is clearly higher than the interest rate prevailing under the flex-rates regime.

Summarizing, the distortion due to sticky interest rates results in an average interest rate \( \bar{R}_{L1} \) which is different from the one required to obtain the flex-rates allocation. It is worth noticing that, in terms of total borrowing an consumption, the effect of the macroeconomic friction is asymmetric. On the one hand, when a positive shock hits the interest rate, debt and consumption are higher than in the flex-rates economy, because interest rates increase by less than they would do in a fully flexible world. On the other hand, when a negative shock hits the economy, debt and consumption are lower than in the flex-rates economy, because interest rates decrease by less than they would do in a fully flexible world.

\(^9\)Notice that the fraction of banks which cannot adjust their rates will keep them at the original level, i.e. \( M R^* \).
3.3 The interaction between the financial friction and the macroeconomic friction

In order to understand how do the macroeconomic and the financial friction interact, Figure 3 displays the optimal borrowing decision ($b_1$) and the crisis probability ($\pi$) in response to a positive shock to the risk free interest rate, with the same calibration as in the previous exercise. On the horizontal axis is the maximum size of the endowment shock. The thick solid line displays the equilibrium when no shock hits the economy (i.e., the same allocation as in Figure 2). The thin line with asterisk markers and the thin line with circle markers display the equilibrium after the shock under flexible rates and sticky rates, respectively.

**Figure 3** Model equilibrium with both frictions - Positive Shock to the interest rate

![Figure 3](image)

**Note.** The thick solid line displays the equilibriums when no shock hits the natural interest rate; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a positive shock hits the risk free rate under flex-rates and sticky-rates, respectively.

The figure shows that, under sticky-rates, the average interest rate in the economy does not increase as much as the risk free rate following a positive shock, prompting consumers to borrow more relative to the efficient level, represented by the flex-rates case (asterisks line). The difference between the circles line and the asterisks line thus quantifies this “overborrowing” in response to positive shocks due to the presence of sticky interest rates.

This type of overborrowing due to sticky-rates is also reflected in the probability of a crisis. The right-hand panel of Figure 3 shows that when a positive shock hits the
economy with flexible interest rates the probability of the constraint to be binding in period 1 is the same as when there is no shock. This is because when consumers choose consumption and borrowing they are facing the efficient level of the interest rate. In contrast, under sticky-rates (circles line), consumers base their optimization on an average interest rate that does not reflect the true one: in the case of a positive shock, their overborrowing results in an increase of the probability of hitting the constraint and, therefore, of generating a crisis the following period.

Notice here that the effect of staggered interest rates setting on the model equilibrium is not symmetric. In fact in the case of a negative shock to the risk free interest rate, sticky interest rate smooth the effects of the financial friction rather than amplifying it. To see that, Figure 4 displays how optimal debt decisions ($b_1$) and crisis probability ($\pi$) vary in response to a negative shock to the risk free interest rate. In contrast to Figure 3, borrowing under sticky-rates (circles line) is lower than in the flex-rates case (asterisks line) and so is the probability of the constraint to be binding in period 1. The mechanism driving this result is simple: the average lending rate now falls by less than the risk free interest rate, restraining borrowing in period 0 and reducing the probability of a crisis in period 1.

Figure 4 Model equilibrium with both frictions - Negative shock to the interest rate

Note. The thick solid line displays the equilibriums when no shock hits the natural interest rate; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a negative shock hits the risk free rate under flex-rates and sticky-rates, respectively.

To summarize, the analysis of the competitive equilibrium of the model brings forth the first positive result of the paper:
Conclusion 1 When the macroeconomic and the financial friction are simultaneously at work, sticky interest rates interact in an asymmetric fashion with the distortionary effects of the financial friction, depending on the sign of the shock hitting the economy. Interest rate stickiness exacerbates the distortion induced by the financial friction conditional on positive shocks to the interest rate, while it dampens the distortionary effects of the financial friction conditional on negative shocks to the interest rate.

4 How to restore efficiency: the planned economy

In this section we consider a social planner who faces the same constraints of atomistic agents but addresses the market failures of our model economy. We will show that a social planner with a macro-prudential instrument (a tax on borrowing) and a monetary policy instrument (the policy interest rate) will address the distortions induced by the credit friction and the macroeconomic friction separately. In particular, monetary policy will focus on the traditional objective of stabilizing inflation and resource allocation; while financial-stability policy will focus on the overborrowing resulting from the pecuniary externality.

We first analyze the distortions generated by the financial friction and by the macroeconomic friction —assuming that they are functioning one at a time— and we compute how the instruments can be designed to restore efficiency. Then, by letting the social planner to address both frictions, we compute the equilibrium and dynamics of the full model: this will constitute the efficient allocation which will be used as a crucial benchmark to understand the main implications of the model.

4.1 Addressing the financial friction: the pecuniary externality

In addition to the financial accelerator mechanism discussed in the previous section, financial frictions also constitute a source of inefficiency because of the presence of pecuniary externalities. In the current setting, the key friction is the presence of an occasionally binding collateral constraint. The pecuniary externality drives a wedge between private and socially optimal outcomes, because atomistic agents do not internalize the effect of their individual decisions on a key market price entering the specification of the aggregate financial friction. More specifically, the social planner unlike the atomistic consumers internalizes that consumption decisions affect the
asset price—as showed by the asset price equation in (7)—which, in turn, affects
the aggregate collateral constraint in (5).

The planner’s problem can be specified as:

$$\max_{b_1, b_2} V = u(b_1) + \mathbb{E} \left[ u(e + b_2 + \pi_1 - b_1 R_{L1}) + y - b_2 R_{L2} \right],$$

where the maximization is subject to the budget constraint (4), the aggregate borrowing constraint (5), and to the pricing rule of the competitive equilibrium allocation:

$$p_1(c_1) = \frac{y}{u'(c_1)},$$

where the asset price, $p_1(c_1)$, is now a function of aggregate consumption. By constraining the social planner problem to the pricing rule of the competitive equilibrium allocation we follow Kehoe and Levine (1993) in the characterization of the constrained efficient outcome.

The corresponding first order conditions are:

$$u'(c_0) = R_{L1} \mathbb{E}[u'(c_1) + \lambda p'(c_1)],$$

$$u'(c_1) = R_{L2} + \lambda(1 - p'(c_1)).$$

By comparing (7) and (16) and noting that $p'(c_1) > 0$, it is clear that the social planner saves more than the competitive agents if there is a positive probability of a binding financial constraints next period ($\mathbb{E} [\lambda] > 0$). This reflects the fact that the social planner internalizes the endogeneity of next period’s asset price and credit constraint to this period’s aggregate saving.

Summarizing, when the constraint never binds, the allocation of resources in the economy is efficient (ignoring the other frictions in the model) and provides the benchmark against which to compare all other allocations. However, when there is a positive probability that constraint binds at time 1, the allocation is not efficient, consumption and borrowing in the decentralized equilibrium are excessive (i.e., there is overborrowing relative to the allocation chosen by a social planner that faces the same constraints, as in Kehoe and Levine (1993), Fernandez-Arias and Lombardo (1998), and Lorenzoni (2008), for example), with a crisis probability that is increasing.

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10 See Arnott et al. (1994) for a discussion.
11 See Benigno et al. (2012) for a discussion of alternative definition of the social planner problem in this context.
in the size of the adverse shock hitting the economy.

As Jeanne and Korinek (2010a) show, a constrained social planner can achieve an efficient allocation in a decentralized economy by imposing a Pigouvian tax on borrowing in period 0, namely $b_t(1 - \tau)$, which is rebated with transfers ($TR$) in a lump-sum fashion. The optimal tax (whose analytical derivation is provided in the appendix) is given by:

$$\tau = E \left[ \lambda p'(m_1) \right].$$

Equation (17) states that whenever there are states in which the borrowing constraint is expected to bind in period 1, both the shadow price of the social planner constraint ($\lambda$) and the derivative $p'(c_1)$ are positive, and hence the social planner impose a positive tax on borrowing in period 0, prompting atomistic agents to consume less and issue less debt in period 0 than under competitive equilibrium.

4.2 Addressing the macro friction: monopolistic competition & interest rate stickiness

As noted above, there are two macroeconomic distortions, one associated with monopolistic competition and a second one associated with staggered interest rate setting. First, monopolistic competition in the banking sector implies an inefficiently low level of consumption, because of higher interest rates relative to perfect competition. Second, time variations in price markups generates an inefficient allocation of resources.\footnote{Notice that, as in the standard New Keynesian literature, the constraints on the frequency of interest rate adjustments in our model economy constitute a source of inefficiency on two different grounds. First, the fact that banks do not adjust their prices continuously implies that the economy’s average markup will vary over time in response to shocks, and will generally differ from the constant frictionless markup $\mathcal{M}$. Second, the presence of staggered interest rate setting implies that relative rates on loans will vary in a way unwarranted by changes in the natural interest rate. Thus, we will generally have $R_{Lt}(i) \neq R_{Lt}(j)$ for any pair of loans $(i, j)$ whose prices do not happen to have been adjusted in the same period. Such relative price distortions will lead, in turn, to different quantities of the different loans being lent, i.e. $b_t(i) \neq b_t(j)$. That outcome violates efficiency conditions. Attaining the efficiency allocation requires that the quantities produced and consumed of all goods are equalized (and so are their prices and marginal costs). Accordingly, markups should be identical across banks and goods at all times, in addition to being constant (and equal to the frictionless markup) on average.}

Notice here that the distortions associated with interest rate stickiness show up only contingent to a shock to the risk free interest rate; that is, when no shock hits the level of the risk free interest rate, the sticky and the flexible interest rate environments do not show any difference.

To unwind the consequences of interest rate stickiness, a social planner would try to affect interest rate in the loans market. Assume that the planner can affect the
interest rate by an additive factor $\psi$, so that the marginal cost for banks would be
given by $R^* + \upsilon + \psi$. Then, the social planner would set:

$$\psi : \bar{R}_{L1} = \mathcal{M}(R^* + \upsilon),$$

which is the efficient level of lending interest rate in the undistorted economy. Solving
this simple equality yields:

$$\psi = \frac{1 - \mu}{\mu} \upsilon.$$

Hence, in response to a positive shock to the risk free rate ($\upsilon > 0$), the social planner
would raise interest rates above the competitive level equilibrium by the factor $\psi > 0$; in contrast, in response to a negative shock to the risk free rate ($\upsilon > 0$), the
social planner would lower interest rates below the competitive level equilibrium by
the factor $\psi < 0$.

Banks that can adjust their interest rates would do that and achieve optimal
allocation; banks that are not allowed to change their interest rates will be not opti-
mizing anyway. But consumers would face the same aggregate interest rate prevailing
without sticky-rates (that is, as in the undistorted economy) and hence make efficient
decisions.

Note here that the presence of a non-trivial price markup implies that the alloca-
tion of the resources in the economy in not Pareto efficient even if interest rates are
flexible. This inefficiency could be eliminated through the suitable choice of subsidy
to interest rate repayments such that:

$$R_{Lt} = \mathcal{M}(1 - \eta_t)R_t.$$

Hence, the optimal allocation can be attained if $\mathcal{M}(1 - \eta_t) = 1$ or, equivalently, by
setting $\eta = \zeta$. By construction, in this case, the equilibrium is efficient.

Notice, however, that the benefits of fostering competition in the banking sector
are ambiguous from a financial stability perspective (see for instance Martinez-Miera
and Repullo (2010), Vives (2011), and Balmaceda et al. (2011)). Given that removing
this additional friction does not change the main properties of the model, in what
follows we solve the model without removing monopolistic competition.
4.3 The efficient allocation

This section graphically displays the equilibrium and the dynamics of our model economy, when both frictions are addressed by a social planner that maximizes the expected utility of consumers (6), subject to their budget constraints (4) and borrowing constraints (5). We assume that the social planner has two instruments to unwind the distortions in the economy: he can affect the interest rate by an additive factor ($\psi$) to address the macroeconomic friction and can impose a tax on debt ($\tau$) to address the financial friction. As noted above, because of the model structure and the time assumptions, the social planner can address the macroeconomic and financial stabilization problems separately, by using the two instruments as explained in the previous sections.

Notice that the sign of $\tau$ is independent of the sign of the shock to the risk free interest rate. The financial friction, in fact, results in overborrowing in period 0 whenever the collateral constraint is binding with positive probability. For the planner is optimal to impose a positive tax on debt ($\tau > 0$) in case of both a positive and a negative shock. In contrast, the sign of $\psi$ depends on the sign of the shock to the risk free interest rate. As showed in the previous sections, when a positive shock hits the economy, the planner has to increase interest rates by $\psi > 0$ to achieve efficiency, while when a negative shock hits the economy the planner has to decrease it by $\psi < 0$ to achieve efficiency.

Consider first a positive shock to the risk free interest rate. Figure 5 displays the response of optimal borrowing decisions ($b_1$) and crisis probability ($\pi$) to the positive shock. As in the previous figures, the thick solid line displays the competitive equilibrium prevailing when no shock hits the risk free rate, the thin line with circle markers and the thin line with asterisk markers display the competitive equilibrium after the positive shock with sticky interest rates and fully flexible interest rates, respectively. Finally, the dashed line displays the equilibrium under a social planner who addresses both the financial and the macroeconomic frictions.

The planner addresses the two frictions separately. On the one hand, the social planner raises interest rates to address the macroeconomic friction. This reduces the overborrowing generated by the interest rate stickiness (circles line) to its constrained-efficient level, represented by the flex-rates equilibrium (asterisks line). On the other hand, the planner imposes a distortionary tax on debt whenever the collateral constraint is expected to be binding with positive probability to address the financial friction. As Figure 5 shows, when the collateral constraint is not expected to bind, the social planner equilibrium and the decentralized equilibrium coincide (i.e., the
**Figure 5** Model equilibrium under social planner with both frictions - Positive shock to the interest rate

<table>
<thead>
<tr>
<th>Max size of the shock (ε)</th>
<th>Level</th>
<th>No shock</th>
<th>Shock (+υ) − No policy</th>
<th>Shock (+υ) − SP Macro</th>
<th>Shock (+υ) − SP Macro &amp; Financial</th>
</tr>
</thead>
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<tr>
<td>0</td>
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<td>0.82</td>
<td>0.82</td>
<td>0.82</td>
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</tr>
<tr>
<td>0.05</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>0.1</td>
<td>0.84</td>
<td>0.84</td>
<td>0.84</td>
<td>0.84</td>
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<td>0.15</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>0.2</td>
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<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
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<td>0.87</td>
</tr>
<tr>
<td>0.3</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
</tr>
</tbody>
</table>

**Note.** The thick solid line displays the equilibriums when no shock hits the natural interest rate; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a positive shock hits the risk free rate under flex-rates and sticky-rates, respectively. The dashed line displays the social planner equilibrium.

optimal tax on debt is zero). In contrast, when the collateral constraint is expected to bind in period 1 with some positive probability, the optimal tax is positive and borrowing under the social planner (dashed line) is lower relative to the competitive equilibrium (circles line).

The right-hand panel of figure 5 illustrates the extent to which the social planner insures the economy against adverse shocks in terms of crisis probability. When the social planner addresses both frictions (dashed line), the probability of a crisis is lower relative to the competitive equilibrium with sticky rates (circles line) and flexible rates (asterisks line) alike. For example, when ε ≃ 0.15, the probability of a crisis is reduced from about 9 percent in an economy where no policy action is taken to about 4 percent.

Consider now the behavior of optimal borrowing decisions ($b_1$) and crisis probability ($\pi$) in response to a negative shock to the risk free interest rate (Figure 6). Again, let’s address first the macroeconomic friction and then the financial friction.

While in response to a positive shock the planner rises interest rates to address the macroeconomic friction ($\psi > 0$) and imposes a positive tax on debt to address the financial friction ($\tau > 0$), in response to a negative shock the planner lowers interest rates ($\psi < 0$) and imposes a positive tax on debt ($\tau > 0$).
Figure 6 Model equilibrium under social planner with both frictions - Negative shock to the interest rate

Note. The thick solid line displays the equilibriums when no shock hits the natural interest rate; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a positive shock hits the risk free rate under flex-rates and sticky-rates, respectively. The dashed line displays the social planner equilibrium.

Remark 1 With two different instruments, namely a macro-prudential tax on borrowing and the policy interest rate, a social planner can address both the financial and the macroeconomic friction and achieve constrained efficiency, independently of the sign of the shock hitting the economy.

Finally, notice that each policy makes use of one instrument to achieve one specific objective. This is regardless of whether a single policy authority is in charge of both monetary and financial-stability policy or whether a policy authority is in charge of monetary policy only and there is a separate authority in charge of financial-stability policy.

5 Monetary policy and financial stability

As we showed in the previous section, to achieve constrained efficiency the planner needs two instruments: a wedge on the lending interest rate faced by households to unwind the distortion generated by staggered interest rates and a Pigouvian tax on borrowing to address the pecuniary externality generated by the financial friction. But what happens when a social planner tries to address both the financial and the macroeconomic friction with a single instrument?
We first show that, when the credit market imperfections are the only frictions in the model, the planner can implement macro-prudential policy by affecting interest rates rather than using the Pigouvian tax. Then, we study the optimal allocation under a social planner with the interest rate as the only instrument, when both frictions are at work simultaneously. Finally, we discuss the implications of the model, with a particular eye on the debate on the role of monetary policy in the run up to the U.S. financial crisis.

5.1 Addressing both frictions with one instrument

Let’s first assume that the only friction in the model economy is given by the credit constraint and that there is only one instrument, namely the policy interest rate. As a direct tax on the level of households’ loans, the social planner can reduce the amount of borrowing undertaken by the agents by increasing lending interest rates. The rationale is the following: at the beginning of period 0, the planner can increase the interest rate by an additive factor $\psi$, as we already assumed above. This would affect banks marginal cost and, therefore, consumers’ borrowing decision through higher debt repayments in period 1.

As we shall see, this policy —rebated with lump sum transfers ($TR$) — has the same effect of the Pigouvian tax analyzed in the previous section. The consumers’ maximization problem becomes:

$$\max_{b_1, b_2, \theta_1, \theta_2} V = u(b_1) + E\left[u(e + b_2 + \pi_1 - b_1M(R^* + \psi) + TR) + y - b_2R_{L2}\right] - \lambda(b_2 - p_1).$$

Again, by equalizing the first order condition with respect to $b_1$ of the decentralized equilibrium and the social planner equilibrium, we can derive the level of $\psi$ which closes the wedge:

$$\left\{ \begin{array}{l}
u'(c_0) = R_{L1}E[u'(c_1) + \lambda p'(c_1)], \\
u'(c_0) = M(R^* + \psi)u'(c_1), \end{array} \right.$$ 

Solving for $\psi$ yields:

$$\psi = E\left[\frac{R^*\lambda p'(c_1)}{u'(c_1)}\right].$$

(18)

Notice that as long as $\lambda$ is different from zero, $\psi$ is positive. This, in turn, implies that whenever the constraint is binding with positive probability, the social planner would raise interest rates so that consumers consume less and issue less debt in period 0, reducing the probability of hitting the constraint in case of an adverse shock next
Remark 2 When the credit constraint is the only friction in the economy and the policy rate is the only instrument, the social planner can address the overborrowing generated by the financial friction and achieve constrained efficiency by increasing interest rates. This allocation is isomorphic to the one obtained with the Pigouvian tax on debt analyzed in the previous section.

Let’s now consider the optimal allocation implemented by a social planner that has only the interest rate as the only policy instrument, when the economy features both frictions and is hit by interest rate shocks. As we showed before, when a positive shock hits this economy both the macroeconomic and the financial friction result in overborrowing in period 0. With the same sequential approach described above, the planner raises interest rates in response to the macroeconomic friction and further raises interest rates to address the financial friction. Therefore, when a positive shock hits the economy, efficiency can be attained with a single instrument.

However, when a negative shock hits the economy, the macroeconomic friction and the financial friction require opposite action on the interest rate. On the one hand, the macroeconomic friction requires a decrease in interest rates. Given that interest rates fall by less than in the flexible rate case, the social planner intervenes to lower interest rates by the factor \( \psi = - (1 - \mu) \frac{\nu}{\mu} < 0 \). On the other hand, the financial friction requires an increase in interest rates independently of the sign of the shock, as showed in equation (18). Therefore, if the interest rate is the only instrument, the social planner would try to lower interest rates to address the macroeconomic friction and, at the same time, to raise interest rate to address the financial friction: not only he will not achieve the efficient allocation, but he will also face a trade off between financial and macroeconomic stability.

Conclusion 2 When macroeconomic and financial frictions are at work simultaneously, if the policy interest rate is the only available instrument, a social planner that aims to achieve both macroeconomic and financial stability may face a policy trade off. In particular, the trade off would only show up when the economy is hit by negative interest rate shocks, because addressing both distortions would require interventions of opposite sign on the interest rate.

Consistently with our finding, in a recent paper Stein (2012) raises the issue of the potential conflict between price stability and financial stability when the policy rate is the only policy instrument. However, building on Goodfriend (2002), he suggests that
the introduction of a new instrument (interest payments on reserves, in his model) would be a way out from the trade off. Woodford (2011) and Svensson (2012a) find similar results in a more standard dynamic New Keynesian model.

5.2 Discussion

This last result has interesting implications for the debate on the role of monetary policy in the run up to the U.S. financial crisis. As we already mentioned in the introduction, under former Chairman Alan Greenspan, the Fed lowered its benchmark rate from 6.5 percent to 1.75 percent in 2000-01, to give an impulse to the economy after the burst of the dot-com bubble and to dissipate deflationary fears. The central bank left the federal funds rate at below 2 percent for about four years, before raising it in quarter-point increments starting from mid 2004.

Against this background, Taylor (2007) supports the idea that the Fed helped inflate U.S. housing prices by keeping rates too low for too long. His main argument starts from the observation that the policy rate was well below of what implied by a standard Taylor rule (see Figure 1): the deviation from a policy framework that was considered standard—at least before the financial crisis—was probably interpreted by market participants as the evidence of a change in the response of policy to changes in inflation. In turn, this prevented long-term interest rates to increase consistently with the monetary policy tightening implemented from 2004 onwards. As a consequence, “those low interest rates were not only unusually low but they logically were a factor in the housing boom and therefore ultimately the bust”.13

Therefore, consistently with the Taylor rule framework, higher interest rates would have reduced both the probability and the severity of the bust the led to the worst recession since the Great Depression. In this section we provide an assessment of this statement, using the implications of our model. In particular, we show that Taylor’s argument is correct only if we make the auxiliary assumption that the central bank is responsible for financial stability—besides the traditional objective of price stability—and it has just one instrument at its disposal, namely the policy rate. However, the argument is no longer valid when the policy authority has two instruments to address the macroeconomic and the financial friction—or, as we showed above, when there are two different policy authorites with one instrument each. In this case, in response to a negative shock, the central bank can slash interest rates as much as needed, without concerns for financial stability which is fully addressed

with the second instrument.

To support our proposition, we bring our stylized model to the data. In particular, we are interested in a negative shock hitting the economy, as the one observed in March 2000 when the dot-com bubble burst. Therefore, we set the beginning of period 0 as the beginning of 2000. We then assume that the economy comes back to its pre-shock level after four years, namely at the beginning of 2004 —supported by the fact that the policy rate has been raised for the first time after 2000 in July 2004— therefore assigning to each time period in our model a length of 4 years. Even if highly stylized, this is a reasonable timing given the observed behavior of economic variables in time frame of interest.

We therefore plot the behavior of the interest rate, as implied by our model, when a negative shock hits the economy and under two alternative assumptions: 1) the policy maker has just one instrument to address both frictions, namely the policy interest rate, and 2) the policy maker has two separate instruments to address the macroeconomic and the financial friction, namely a tax on borrowing and the policy interest rate. Figure 7 displays the interest rate behavior under this two alternative assumptions.

**Figure 7** Alternative path of the interest rate under different assumptions about the number of available instruments

![Graph showing alternative paths of interest rate](image)

**Note.** *Flex* displays the behavior of interest rates in a fully flexible framework; *SP* displays the path implied by a social planner addressing both the macroeconomic and the financial frictions with one or two instruments, respectively.

As illustrated in the previous section and stated in Remark 2, if the policy rate is the only instrument and conditional on negative shocks to the economy, the social planner faces a trade off. The planner would try to lower interest rates to address the
macroeconomic friction and restore the average lending rate that would prevail in the absence of interest rate stickiness; but would also try to raise interest rates to address the financial friction and reduce the overborrowing generated by the credit externality. The trade off is illustrated by the left-hand panel of Figure 7. Assuming that the weight attached to macroeconomic and financial stability is the same, interest rates would fall by less than in the flex-rate case. Therefore, under this assumption, our model supports Taylor’s argument in the sense that there is a scope for the central bank to keep interest rates higher than the flex-price case, to avoid overborrowing, excessive asset prices and to reduce the probability of a crisis if the economy is hit by a negative shock next period.

The results are different when the social planner can address both the financial and the macroeconomic friction with two different instruments. As noted above, this case is equivalent to the case in which there are two separate and independent policy authorities, as for example a central bank with the objective of price stability and a financial regulator with the objective of financial stability. As stated in Remark 1, such a planner can achieve constrained efficiency with two independent policy actions, despite the sign of the shock. Therefore, once the overborrowing generated by the financial friction is addressed with a Pigouvian tax of the type analyzed above, it is optimal for the central bank to lower interest rates in order to restore the flex-rate allocation.

6 Conclusions

If the policy rate is the only available tool, achieving the monetary policy objectives and maintaining financial stability entails a trade off that should be taken into account by the policy authority. Specifically, the use of the policy interest rate as the only instrument to address both macroeconomic and financial frictions might lead to poorer outcomes —relative to a case in which two instruments are available— for macroeconomic and financial stability alike. Normally, however, other instruments are at policy makers’ disposal in order to achieve and maintain financial stability. Our model shows that, when two instruments are available, the trade off disappears. This, in turn, implies that the same monetary policy stance as the one adopted by the Fed during the 2002-06 period, accompanied by stronger regulation and supervision of the financial system, might have been more effective in reducing the likelihood and the severity of the crisis —relative to a tighter monetary policy stance with the same financial supervision and regulation observed during the 2002-06 period.

This has important implications concerning the debate on the role played by
monetary policy for the stability of the financial system in the run up to the global financial crisis. In a series of work Taylor (2007, 2010) suggest that higher interest rates in the 2002-2006 period would have reduced both the probability and the severity of the crisis. Our findings support this argument only if we make the auxiliary assumption that the policy authority —addressing all distortions present in our model— has just one instrument at its disposal, namely the policy rate. In contrast, when the policy authority has two different instruments, interest rates can be lowered as much as needed without concerns for financial stability, supporting the view of Bernanke (2010) that additional policy tools were needed to prevent the global financial crisis.

Hence, our simple model suggests that it is not the monetary function of the Federal Reserve system alone that can be blamed for the crisis. In contrast, the weak link in the policy framework in the run up to the crisis seems not to be an excessively lax monetary policy stance, but rather the absence of a second policy instrument aimed at preserving systemic financial stability.
References


A Appendix

Three appendices show the derivation of loans demand in the Dixit-Stiglitz framework, the derivation of the optimal Pigouvian tax to unwind the distortion generated by the financial friction, and the numerical solution used to solve the model when the collateral constraint is only occasionally binding.

A.1 Derivation of Loans Demand

In a Dixit-Stiglitz framework, household $i$, in order to obtain a loan of a given size $b_t(i)$, needs to take out a continuum of loans $b_t(i,j)$ from all existing banks $j$, such that:

$$b_t(i) \leq \left( \int_0^1 b_t(i,j) \frac{\zeta - 1}{\zeta} dj \right)^{\frac{1}{\zeta-1}} \quad (A.1)$$

where $\zeta > 1$ is the elasticity of substitution between differentiated loans (or banking services, in general).

Demand by household $i$ seeking an amount of real loans equal to $b_t(i)$ can be derived from minimizing over $b_t(i,j)$ the total repayment due to the continuum of banks $j$:

$$\min_{b_t(i)} R_{Lt}b_t(i) = \min_{b_t(i,j)} \int_0^1 R_{Lt}(j)b_t(i,j) dj$$

subject to (A.1). The first order condition reads:

$$R_{Lt}(j) - \lambda \frac{\zeta}{\zeta - 1} \left( \int_0^1 b_t(i,j) \frac{\zeta - 1}{\zeta} dj \right)^{\frac{\zeta - 1}{\zeta - 1} - 1} \frac{\zeta - 1}{\zeta} b_t(i,j) = 0 \quad (A.2)$$

where $\lambda$ is the Lagrangian multiplier associated with the minimization problem. First, rearrange using (A.1):

$$R_{Lt}(j) = \lambda b_t(i) \frac{\zeta}{\zeta - 1} b_t(i,j)^{\frac{1}{\zeta}} \quad (A.3)$$

Then, take both sides to the power of $1 - \zeta$ to get:

$$R_{Lt}(j)^{1-\zeta} = \lambda^{1-\zeta} b_t(i) \frac{1-\zeta}{\zeta} b_t(i,j)^{-\frac{1-\zeta}{\zeta}} \quad (A.4)$$

and integrate over all $j$ to get:

$$\lambda = \left( \int_0^1 R_{Lt}(j)^{1-\zeta} dj \right)^{\frac{1}{1-\zeta}} \equiv R_{Lt} \quad (A.5)$$

The Lagrangian multiplier is the aggregated interest rate for loans. Finally, plugging (A.5) into (A.4) and aggregating over symmetric households, the minimization problem yields downward-sloping loans demand curves of the kind:

$$b_t(j) = \left( \frac{R_{Lt}(j)}{R_{Lt}} \right)^{-\zeta} b_t \quad (A.6)$$
A.2 Derivation of the Pigouvian Tax

As Jeanne and Korinek (2010a) show, a constrained social planner can achieve an efficient allocation in a decentralized economy by imposing a Pigouvian tax on borrowing in period 0, namely $b_1(1 - \tau)$, which is rebated with transfers ($TR$) in a lump-sum fashion. To see this, let’s write the maximization problem as:

$$\max_{b_1, b_2} V = u((1 - \tau)b_1 + TR) + \mathbb{E}[u(e + b_2 + \pi_1 - b_1R_{L1}) + y - b_2R_{L2}] - \lambda(b_2 - p_1).$$

The optimal tax is computed by comparing the FOC($b_1$) in the decentralized and the social planner problems:

$$\begin{cases} u'(c_0) - u'(c_1)R_{L1} - \lambda p'(m_1)R_{L1} = 0, \\ u'(c_0) \cdot (1 - \tau) - u'(c_1)R_{L1} = 0. \end{cases}$$

Solving for $\tau$ yields:

$$\tau = \mathbb{E}\left[\frac{\lambda p'(m_1)}{u'(c_1)}\right]. \quad (A.7)$$

A.3 Numerical Solution

Preliminaries. The first order conditions of the competitive equilibrium (CE) are:

$$\begin{cases} FOC(b_1) : u'(c_0) = \mathbb{E}[R_{L1}u'(c_1)], \\ FOC(b_2) : u'(c_1) = R_{L2} + \lambda, \\ FOC(\theta_2) : p_1 = \frac{y}{u'(c_1)}. \end{cases}$$

The presence of the Lagrange multiplier shows that consumers are aware of the financial friction. In fact, they know that, in case of an adverse shock to the endowment, they might not be able to borrow as much as they would like. Therefore, whenever the collateral constraint is expected to bind at time 1 (i.e., whenever $\mathbb{E}[\lambda] > 0$), they reduce their optimal amount of consumption at time 0 and 1.

When the economy is not constrained ($\lambda = 0$) the model has the following close form solution:

$$\begin{cases} u'(c_1) = R_{L2}, \\ u'(c_0) = \mathbb{E}[R_{L2}R_{L1}], \\ p_1 = \frac{y}{R_{L2}}. \end{cases} \quad \Rightarrow \quad \begin{cases} c_1^* = (R_{L2})^{-\frac{1}{\delta}}, \\ c_0^* = b_1^* = (R_{L2}R_{L1})^{-\frac{1}{\delta}}, \\ p_1 = \frac{y}{R_{L2}}. \end{cases}$$
Moreover, the collateral constraint must hold for the optimal values derived above:\(^{14}\)

\[
\frac{b_2}{c_1^* + b_1^*R_{L1} - e} \leq \frac{p_1}{\frac{y}{R_{L2}}},
\]

which we can rewrite as:

\[
e \geq e^* = c_1^* + b_1^*R_{L1} - \frac{y}{R_{L2}} = (R_{L2})^{-\frac{1}{\sigma}} + (R_{L2}R_{L1})^{-\frac{1}{\sigma}} R_{L1} - \frac{y}{R_{L2}}.
\]

That is, whenever the endowment is above a certain threshold, the economy is not constrained.

On the other hand, when the economy is constrained (\(e < e^*\)) the collateral constraint is binding and consumers would like to borrow \(b_2 > p_1\). Given that this is not possible, consumers will borrow as much as they can, trying to maximize their consumption at time 1. Therefore, the collateral constraint will bind with equality \(b_2 = p_1\),

\[
c_1 + b_1R_{L1} - e = \frac{y}{\psi(c_1)}
\]

(A.8)

Therefore, depending whether the constraint is binding or not, we can express borrowing at time 0 as:

\[
b_1 = \begin{cases} 
(R_{L2}R_{L1})^{-\frac{1}{\sigma}} & e \geq e^* \\
\frac{ye_1^2 - c_1 + e}{R_{L1}} & e < e^*
\end{cases}
\]

(A.9)

Finally we assume that the endowment is stochastic and follows a uniform distribution \(e \sim U(\bar{e} - \varepsilon, \bar{e} + \varepsilon)\).

**Definition of shocks and states of nature.** To be able to solve the model we need to calibrate three parameters: \(y, \bar{e}, \) and \(\varepsilon\). In particular, we will consider calibrations such that 1) the economy may be constrained for sufficiently large negative shocks but 2) would not be constrained in the absence of uncertainty.

First, we want a condition that is necessary and sufficient for the economy to be constrained with some probability, when \(e \sim U(\bar{e} - \varepsilon, \bar{e} + \varepsilon)\). Let’s reason the other way round: we already showed that the economy is indeed unconstrained in period 1 if and only if:

\[
e \geq e^* = c_1^* + b_1^*R_{L1} - \frac{y}{R_{L2}}.
\]

When \(e\) is stochastic, for the economy to be unconstrained, the above inequality must hold for all possible realizations of \(e\) (in particular the adverse realizations). In other words it must be the case that:

\[
e - \varepsilon \geq c_1^* + b_1^*R_{L1} - \frac{y}{R_{L2}},
\]

\[
\bar{e} \geq c_1^* + b_1^*R_{L1} - \frac{y}{R_{L2}} + \varepsilon.
\]

\(^{14}\)Note here that I am assuming that profits are realized at the end of the period so that they have no effect on the borrowing constraint.
Therefore:

- when $\bar{e} < c^* + b^1 R_{L1} - \frac{y}{R_{L2}} + \varepsilon$ there exists a non-zero probability that the constraint binds.

That is, if the endowment is too high relative to the shock, the constraint will never be binding.

Second, when there is no uncertainty around the realizations of $e$ (i.e., $\varepsilon = 0$ and $\bar{e} = e$) the constraint is not binding in period 1 if and only if $e = \bar{e} \geq e^*$, that is:

$$\bar{e} \geq c^* + b^1 R_{L1} - \frac{y}{R_{L2}}.$$

Therefore:

- with no uncertainty, when $\bar{e} \geq c^* + b^1 R_{L1} - \frac{y}{R_{L2}}$ the constraint never binds

Summarizing we choose an $\bar{e}$ such that would not be constrained in the absence of uncertainty but it economy may be constrained for sufficiently large negative shocks:

$$(R_{L2})^{-\frac{1}{\rho}} + (R_{L2} R_{L1})^{-\frac{1}{\rho}} R_{L1} - \frac{y}{R_{L2}} \leq \bar{e} < (R_{L2})^{-\frac{1}{\rho}} + (R_{L2} R_{L1})^{-\frac{1}{\rho}} R_{L1} - \frac{y}{R_{L2}} + \varepsilon.$$

This implies that there will be a threshold for the size of the shock ($\varepsilon^b$) above which the collateral constraint will start to be binding. Notice that the constraint is binding for adverse (negative) realizations of the shock. That is, the collateral constraint would be binding for realizations of $e$ in the interval $[\bar{e} - \varepsilon, \bar{e} - \varepsilon^b]$. The level of $\varepsilon^b$ can be easily computed as:

$$\varepsilon^b = \bar{e} - e^*.$$

**Competitive equilibrium.** We will find numerical values for consumption at time 1 ($c_1$) by using is the Euler equation $FOC(b_1)$, which gives us an optimal relation between consumption at time 0 and consumption at time 1. In order to be able to solve this equation we need 1) to find an expression for borrowing as a function of consumption for both constrained and unconstrained states, as we already did in equation (A.9); and 2) to weight those states for their probability.

First, combining $FOC(b_1)$, the budget constraint, and the expression for $b_1$ derived earlier in equation (A.9) we get:

$$\left\{ \begin{array}{lcl} E[b_1^{-\rho}] &=& R_{L1} E[c_1^{-\rho}], \\ b_1 &=& \begin{cases} (R_{L2} R_{L1})^{-\frac{1}{\rho}} & e \geq e^*, \\ \frac{y e^1 - c_1 + \varepsilon}{R_{L1}} & e < e^*. \end{cases} \end{array} \right.$$  

Second, given the assumed distribution for the endowment, the level of consumption in period 1 will be given by:

$$Pr(e < e^*) \cdot [b_1^{-\rho}]^{\text{binding}} + Pr(e \geq e^*) \cdot [b_1^{-\rho}]^{\text{non-binding}} = R_{L1} c_1^{-\rho}.$$  

---

15Remember that $c_0 = b_1$ from the budget constraint.
The LHS of the previous equation can be expressed as follows:\(^{16}\):

\[
\mathbb{E}[b_1^{-\varrho}] = \frac{1}{2\varepsilon} \int_{\bar{e}-\varepsilon}^{e-\varepsilon} \left( \frac{yc_1^\varrho - c_1 + e}{R_{L1}} \right)^{-\varrho} d\varepsilon + \frac{1}{2\varepsilon} \int_{e-\varepsilon}^{\bar{e}+\varepsilon} R_{L2} R_{L1} d\varepsilon = \\
= \frac{1}{2\varepsilon} \int_{\bar{e}-\varepsilon}^{e-\varepsilon} \left( \frac{yc_1^\varrho - c_1 + e}{R_{L1}} \right)^{-\varrho} d\varepsilon + \frac{R_{L2} R_{L1}}{2\varepsilon} \left[ \varepsilon^{\bar{e}+\varepsilon} \right] \\
= \frac{1}{2\varepsilon} \left[ R_{L1} \left( \frac{yc_1^\varrho - c_1 + e}{R_{L1}} \right)^{-\varrho+1} \right]_{\bar{e}-\varepsilon}^{e-\varepsilon} + \frac{R_{L2} R_{L1}}{2\varepsilon} \left[ \varepsilon + \varepsilon^{\bar{e}} \right] \\
= \frac{R_{L1}^\varrho}{2\varepsilon (1 - \varrho)} \left( yc_1^\varrho - c_1 + e \right)^{-\varrho+1} \left[ \varepsilon + \varepsilon^{\bar{e}} \right] + \frac{R_{L2} R_{L1}}{2\varepsilon} \left[ \varepsilon + \varepsilon^{\bar{e}} \right].
\]

The following equation can be solved numerically to obtain the competitive equilibrium level of consumption at time 1:

\[
\begin{align*}
\text{LHS} &= \text{RHS} \\
\mathbb{E}[b_1] &= \mathbb{E} \left[ (R_{L1} c_1^{-\varrho})^{-\frac{1}{\varrho}} \right].
\end{align*}
\]

**Social planner.** The social planner problem is solved with the same strategy. The first order conditions are:

\[
\begin{align*}
\text{FOC}(b_1) : \quad & u'(c_0) = R_{L1} \mathbb{E}[u'(c_1) + \lambda p'(c_1)], \\
\text{FOC}(b_2) : \quad & u'(c_1) = R_{L2} + \lambda (1 - p'(c_1)), \\
\text{FOC}(\theta_2) : \quad & p_1 = \frac{y}{u'(c_1)}. 
\end{align*}
\]

First we have to find an expression for \(p'(c_1)\). From \(\text{FOC}(\theta_2)\) we get:

\[
p(c_1) = \frac{y}{u'(c_1)} = yc_1^\varrho,
\]

and computing the derivative:

\[
p'(c_1) = \frac{\partial (yc_1)}{\partial c_1} = yc_1^{\varrho-1}.
\]

Notice that the \(p'(c_1)\) is positive and decreasing. Notice also that, by definition, the Lagrange multiplier (\(\lambda\)) is positive only when the constraint is binding. By looking

\(^{16}\text{Suppose that } X \text{ has the } U(a, b) \text{ distribution. Then the } n^{th} \text{ moment of } X \text{ is given by } \mathbb{E}[X^n] = \frac{1}{b-a} \int_a^b x^ndx.\)
at \( FOC(b_1) \) of the social planner problem, we can state that the planner limits over-borrowing. In fact, \( u'(c_1)^{SP} > u'(c_1)^{CE} \) which implies that consumption and, therefore, borrowing at time 1 are lower relative to the competitive equilibrium. On the other hand, the planner increases consumption in period 1: given that \( p'(c_1) > 0 \) from \( FOC(b_2) \) we see that \( u'(c_1)^{SP} < u'(c_1)^{CE} \).

We also need a value of \( \lambda \). Notice that the Lagrange multiplier of the social planner is numerically different from the one of the competitive equilibrium problem. In fact, from \( FOC(b_2) \) we get

\[
\lambda = \frac{c_1 - R_{L2}}{1 + y}.
\]

Combining these two results we can compute:

\[
\lambda p'(c_1) = \begin{cases} 
0 & e \geq e^*, \\
\frac{\varrho y}{1 + y} \left( c_1^{-1} - R_{L2}c_1^{-1} \right) & e < e^*.
\end{cases}
\]

We can now solve for the level of \( c_1 \). The \( FOC(b_1) \) can be written:

\[
E \left[ b_1^{-\varrho} \right] = R_{L1} E \left[ c_1^{-\varrho} + \lambda p'(c_1) \right].
\]

The LHS has already been computed before. The RHS is:

\[
\frac{R_{L1}}{2\varepsilon} \int_{\bar{c} - \varepsilon}^{\varepsilon + \varepsilon} \left( c_1^{-\varrho} + \frac{\varrho y}{1 + y} \left( c_1^{-1} - R_{L2}c_1^{-1} \right) \right) de + \frac{R_{L1}}{2\varepsilon} \int_{\bar{c} - \varepsilon}^{\varepsilon + \varepsilon} c_1^{-\varrho} de,
\]

\[
\frac{R_{L1}}{2\varepsilon} \left[ \left( c_1^{-\varrho} + \frac{\varrho y}{1 + y} \left( c_1^{-1} - R_{L2}c_1^{-1} \right) \right) (\varepsilon - \varepsilon^b) + c_1^{-\varrho} (\varepsilon + \varepsilon^b) \right],
\]

\[
\frac{R_{L1}}{2\varepsilon} \left[ \frac{\varrho y}{1 + y} \left( c_1^{-1} - R_{L2}c_1^{-1} \right) \right] (\varepsilon - \varepsilon^b) + 2c_1^{-\varrho} \varepsilon.
\]

Therefore, the following equation can be solved numerically to find the optimal consumption at time 1:

\[
\begin{align*}
\text{LHS} & = \text{RHS} \\
\text{LHS} & = \frac{R_{L1}^2}{2\varepsilon (1 - \theta)} \left[ (yc_1^\varrho - c_1 + \bar{\varepsilon} - \varepsilon^b)^{-\varrho + 1} - (yc_1^\varrho - c_1 + \bar{\varepsilon} - \varepsilon)^{-\varrho + 1} \right] + \frac{R_{L2}R_{L1}}{2\varepsilon} \left[ \varepsilon + \varepsilon^b \right] \\
\text{RHS} & = \frac{R_{L1}}{2\varepsilon} \left[ \frac{\varrho y}{1 + y} \left( c_1^{-1} - R_{L2}c_1^{-1} \right) \right] (\varepsilon - \varepsilon^b) + 2c_1^{-\varrho} \varepsilon.
\end{align*}
\]

Finally, one can derive the optimal expression for borrowing at time 1 from the social planner \( FOC(b_1) \):

\[
b_1 = \left( R_{L1} E \left[ c_1^{-\varrho} + \lambda p'(c_1) \right] \right)^{-\frac{1}{\varrho}}.
\]

**Probability of the constraint to be binding.** Is given by:

\[
Pr = \frac{1}{2\varepsilon} \int_{\bar{\varepsilon} - \varepsilon}^{\varepsilon - \varepsilon^b} de = \frac{1}{2\varepsilon} (\varepsilon - \varepsilon^b).
\]
Notice however that this formulation does not take into account that, when the constraint is expected to bind with positive probability, the social planner reduces borrowing and therefore also reduces the probability of a crisis. Noticing that $e - e_b = c_1 + b_1 R_{L1} - \frac{y}{R_{L2}}$, we can compute

$$Pr = \frac{1}{2\varepsilon} \int_{e-e_b}^{e-e_b} de = \frac{1}{2\varepsilon} \left( c_1 + b_1 R_{L1} - \frac{y}{R_{L2}} - e + \varepsilon \right).$$